Test

Computers forbidden — Books, lecture notes and personal notes allowed

N.B. The length of the test and the number of exercices should not frighten the students nor be interpreted as being part of the difficulty of the test. On the contrary, it is an opportunity for someone who would be blocked on a question to solve another one.

In the following, A denotes the alphabet $A = \{a, b\}$, $\mathbb{N}$, the set of non negative integers. If w is in A^*, we denote by $|w|_a$ the number of a's in w.

1 .— Finite image relations.

Recall that a relation α is said to be *finite image* if $\mathsf{Im}\,\alpha$ is a finite set (and not if the image $\alpha(w)$ is finite for every w).

Show that a finite image functional rational relation is sequential.

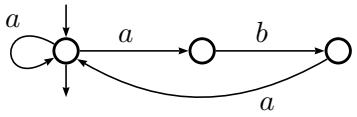
2 .— Commutative image.

Let $\alpha\colon A^* \to \mathbb{N}^2$ the *commutative image* map, i.e. $\alpha(w) = (|w|_a, |w|_b)$.

Show that the equivalence map of α, i.e. the relation $\alpha^{-1} \circ \alpha\colon A^* \to A^*$ which associates with every word w of A^* all the words of A^* which have the same number of a's and the same number of b's as w, is not a rational relation.

3 .— Coding and deciphering.

(i) Build the Schützenberger covering of the automaton below.

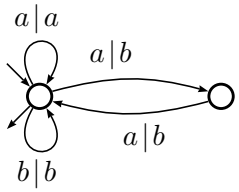


(ii) Let $\alpha\colon \{x, y\}^* \to \{a, b\}^*$ be the morphism defined by : $\alpha(x) = a$ et $\alpha(y) = aba$. Show that α is *injective* (hence the relation α^{-1} is fonctional).

(iii) Give a (finite) sequential transducer that realizes α^{-1}.

4 .— **Factor replacing.**

(i) Let $\alpha\colon A^* \to A^*$ be the relation realized by the synchronous transducer below.



(a) What is the image of the word $abaabb$ by α ?

(b) Describe the relation α.

(c) Give a transducer which realizes $\alpha \circ \alpha$.

(ii) Let $\beta\colon A^* \to A^*$ be the (functional) relation which replace *every* factor ab of a word by a factor ba (which does not prevent the result to contain still factors ab). For instance : $\beta(abaabb) = baabab$.
Give a synchronous transducer which realizes β. [*Hint:* 3 *states*]

(iii) (a) Give a sequential transducer which realizes β. [*Hint:* 2 *states*]

(b) Give a sequential transducer which realizes $\beta \circ \beta$.