MASTER PARISIEN DE RECHERCHE EN INFORMATIQUE

COURS 2-16 “MODÉLISATION PAR AUTOMATES FINIS”

11 March 2011 — Exam

Books and printed notes forbidden — Personal notes allowed

The length of the text and the number of exercises should not frighten you nor be understood as part of the difficulty of the exam but as the opportunity given to a student who is ‘stuck’ on a question to try to solve another one.

Reduction et sequentialisation of weighted automata

I. $\mathbb{Q}$-automata

Let $\mathcal{A}_1$ be the $\mathbb{Q}$-automaton on $\{a\}^*$ shown at Figure 1 (the unique letter a of the alphabet is not shown on the transitions of the figure) and s_1 the series it realises.

1 .— Give a reduced representation of s_1.

2 .— Compute $\langle s_1, a^5 \rangle$, $\langle s_1, a^6 \rangle$.

3 .— Is s_1 a sequential series? (that is, is it realised by a row-monomial representation?)

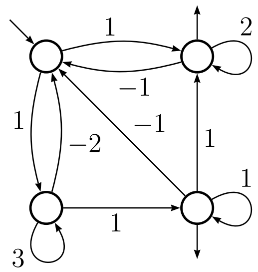

Figure 1: The $\mathbb{Q}$-automaton $\mathcal{A}_1$

II. $\mathcal{M}$-automata

Let $\mathcal{E}_1$ and $\mathcal{E}_2$ be the automata over $\{a\}^*$ shown at Figure 2 (a) when the weight semirings are supposed to be $\langle \mathbb{N}, \mathsf{min}, + \rangle$ and $\langle \mathbb{N}, \mathsf{max}, + \rangle$ respectively and t_1 and t_2 the series that they realise. Accordingly, let $\mathcal{F}_1$ and $\mathcal{F}_2$ be the automata over $\{a\}^*$ shown at Figure 2 (b) when the weight semirings are supposed to be $\langle \mathbb{N}, \mathsf{min}, + \rangle$ and $\langle \mathbb{N}, \mathsf{max}, + \rangle$ respectively and u_1 and u_2 the series that they realise.

4 .— Give a formula for $\langle t_1, a^n \rangle$ and for $\langle t_2, a^n \rangle$. Are these series sequential?

5 .— Same questions for u_1 and u_2.

6 .— Apply the general sequentialisation procedure to these four automata. Comment.

(a) The automata $\mathcal{E}_1$ and $\mathcal{E}_2$ (b) The automata $\mathcal{F}_1$ and $\mathcal{F}_2$

Figure 2: Four tropical automata

Automata with bounded ambiguity and the Schützenberger covering

Let us recall that the Schützenberger covering $\mathcal{S}$ of an automaton $\mathcal{A}$ is the accessible part of the product of $\mathcal{A}$ by its determinisation $\widehat{\mathcal{A}}$. The projection of $\mathcal{S}$ onto $\mathcal{A}$ is a covering, the one onto $\widehat{\mathcal{A}}$ is a locally co-surjective morphism.

Definition 1 Let $\mathcal{S}$ be the Schützenberger covering of an automaton $\mathcal{A}$.
We call *concurrent transition set* of $\mathcal{S}$ a set of transitions which
- (a) have the same destination (final extremity),
- (b) are mapped onto the same transition of $\widehat{\mathcal{A}}$.

Two transitions of $\mathcal{S}$ are called *concurrent* if they belong to the same concurrent transition set.

In the sequel, $\mathcal{A}$ is an automaton, $\widehat{\mathcal{A}}$ its determinisation, and $\mathcal{S}$ its Schützenberger covering. We also set the folllowing definition:

Definition 2 An automaton $\mathcal{A}$ over A^* is of *bounded ambiguity* if there exists an integer k such that every word w in $|\mathcal{A}|$ is the label of at most k distinct computations. The smallest such k is the *ambiguity degree* of $\mathcal{A}$.

7 .— Compute the Schützenberger covering of the automaton $\mathcal{B}_1$ of the Figure 3 (*It should have 8 states*).

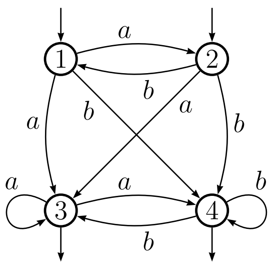

Figure 3: The automaton $\mathcal{B}_1$

8 .— What can be said of an automaton whose Schützenberger covering contains no concurrent transitions?

9 .— Show that there exists a computation in $\mathcal{S}$ which contains two transitions of the same concurrent transition set if, and only if, there exists a concurrrent transition which belongs to a circuit.

10 .— Let $p \xrightarrow{a} s$ and $q \xrightarrow{a} s$ be two concurrent transitions of $\mathcal{S}$ and

$$c \;:=\; \xrightarrow[\mathcal{S}]{} i \xrightarrow[\mathcal{S}]{x} p \xrightarrow[\mathcal{S}]{a} s \xrightarrow[\mathcal{S}]{y} q \xrightarrow[\mathcal{S}]{a} s \xrightarrow[\mathcal{S}]{z} t \xrightarrow[\mathcal{S}]{}$$

a computation of $\mathcal{S}$ where i is an initial state and t a final state. Show that $w = x\,a\,y\,a\,z$ is the label of at least two computations of $\mathcal{A}$.

11 .— Prove that an automaton $\mathcal{A}$ is of bounded ambiguity if, and only if, no concurrent transition of its Schützenberger covering belongs to a circuit.

12 .— Check that $\mathcal{B}_1$ is of bounded ambiguity.

13 .— Give a bound on the ambiguity degree of an automaton as a function of the cardinals of the concurrent transition sets of its Schützenberger covering.

Compute that bound in the case of $\mathcal{B}_1$.

14 .— Infer from the above the complexity of an algorithm which decide if an automaton if of bounded ambiguity.